MTH 1420, SPRING 2012
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SECTION 5.9: APPROXIMATE INTEGRATION

HW: 18 (only need to do M_8 and M_n)

Practice: 17

1. Introduction

In Lab #4 we look at approximate integration. In this section, we will introduce two other techniques of approximate integration and look at how we can get an estimate of how good our approximations are (by looking at estimates of the *error* for a given approximation).

2. The Trapezoidal Rule and Simpson's Rule

One way of approximating the area under a curve is to do something like a Riemann sum, but use <u>trapezoids</u> to approximate the area. The idea is this:

Theorem 2.1. *(Trapezoidal Rule)*

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

where $\Delta x = (b-a)/n$ *and* $x_i = a + i\Delta x$. *The approximation obviously gets better the larger value of* n *you choose.*

Another way of approximating the area under the integral is to use parabolas. The idea is this:

Theorem 2.2. *(Simpson's Rule)*

$$\int_a^b f(x)\,dx \approx S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

where n *is even,* $\Delta x = (b-a)/n$ *and* $x_i = a + i\Delta x$.

We will not be using the Trapezoidal Rule or Simpson's Rule for approximations, since the Midpoint Rule is good enough for our purposes.

3. Error Estimates

One important aspect of approximating integrals is to know how close you are to the real answer. If you want to be within 0.001 of the actual answer, how large do you have to choose n to be when you approximate? It turns out that there are bounds for the error of all of the methods we have looked at. The one we will use is the error bound for the Midpoint Rule:

Theorem 3.1. *(Error bound for the Midpoint Rule) Suppose $|f"(x)| \leq K$ for all x between a and b (that is, the limits of integration for the area you want to approximate). Then the error for the Midpoint Rule is*

$$E_M \leq \frac{K(b-a)^3}{24n^2}$$

Example 1. (a) Find the approximation M_{10} for $\int \ln(x^3+2)\,dx$.

(b) Estimate the error in the approximation from part (a).

(c) How large do we have to choose n so that the approximation M_n is accurate to within 0.0001?